Flood Routing

Continuity equation

Hydrologic Routing

Hydraulic Routing

Momentum Equation

- Simulate the movement of water through a channel
- Used to predict the magnitudes, volumes, and temporal patterns of the flow (often a flood wave) as it translates down a channel.
- 2 types of routing : hydrologic and hydraulic.
- both of these methods use some form of the continuity equation.

Continuity Equation

Continuity equation

Hydrologic Routing

Hydraulic Routing

Momentum Equation

•The change in storage (dS) equals the difference between inflow (I) and outflow (O) or :

$$\frac{dS}{dt} = I - O$$

•For open channel flow, the continuity equation is also often written as :

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q$$

A = the cross-sectional area,
Q = channel flow, and
q = lateral inflow

Hydrologic Routing

Continuity equation

Hydrologic Routing

Hydraulic Routing

Momentum Equation

- Methods combine the continuity equation with some relationship between storage, outflow, and possibly inflow.

- These relationships are usually assumed, empirical, or analytical in nature.

- An of example of such a relationship might be a stage-discharge relationship.

Use of Manning Equation

Continuity equation

Hydrologic Routing

Hydraulic Routing

Momentum Equation

- Stage is also related to the outflow via a relationship such as Manning's equation

$$Q = \frac{1}{n} A R_h^{2/3} S_f^{1/2}$$

Hydraulic Routing

Continuity equation

Hydrologic Routing

Hydraulic Routing

Momentum Equation

- Hydraulic routing methods combine the continuity equation with some more physical relationship describing the actual physics of the movement of the water.

- The momentum equation is the common relationship employed.

- In hydraulic routing analysis, it is intended that the dynamics of the water or flood wave movement be more accurately described

Momentum Equation

- Expressed by considering the external forces acting on a control section of water as it moves down a channel

$$\frac{\partial v}{\partial t} + V\frac{\partial v}{\partial x} + \frac{g}{A}\frac{\partial(\overline{y}A)}{2x} + \frac{vg}{A} = g(S_o - S_f)$$

- Henderson (1966) expressed the momentum equation as :

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t}$$

Combinations of Equations

Continuity equation

Hydrologic Routing

Hydraulic Routing

Momentum Equation

- Simplified Versions :

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t}$$

Unsteady -Nonuniform

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x}$$

Steady - Nonuniform

$$S_f = S_o - \frac{\partial y}{\partial x}$$

Diffusion or noninertial

$$S_f = S_o$$

Kinematic

Routing Methods

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- Modified Puls

- Kinematic Wave

- Muskingum

- Muskingum-Cunge

- Dynamic

Modified Puls

- The modified puls routing method is probably most often applied to reservoir routing

- The method may also be applied to river routing for certain channel situations.

- The modified puls method is also referred to as the storage-indication method.

- The heart of the modified puls equation is found by considering the finite difference form of the continuity equation.

Modified Puls

$$\frac{I_1 + I_2}{2} - \frac{(O_1 + O_2)}{2} = \frac{S_2 - S_1}{\Delta t}$$ Continuity Equation

$$I_1 + I_2 + \left(\frac{2 S_1}{\Delta t} - O_1 \right) = \frac{2 S_2}{\Delta t} + O_2$$ Rewritten

• The solution to the modified puls method is accomplished by developing a graph (or table) of O -vs- $[2S/\Delta t + O]$. In order to do this, a stage-discharge-storage relationship must be known, assumed, or derived.
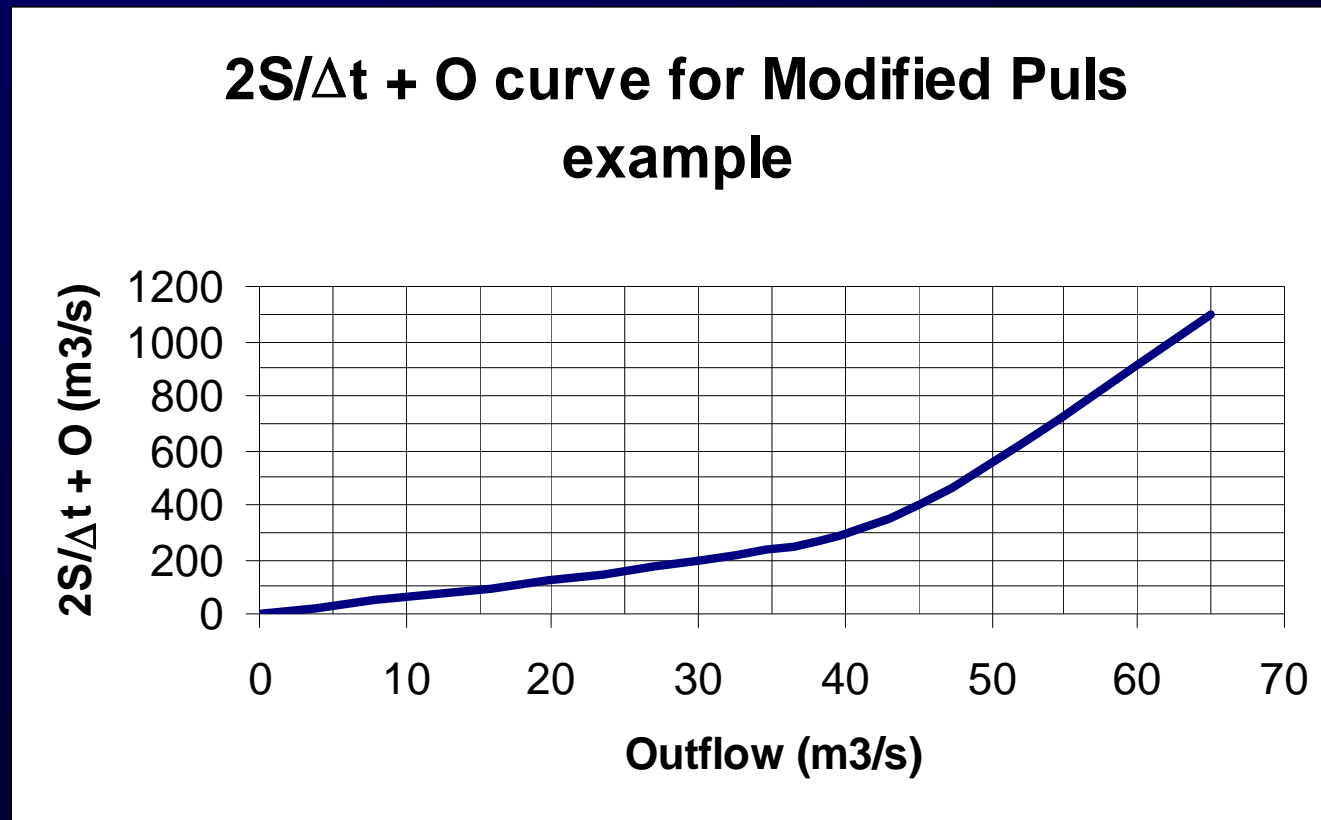
Modified Puls Example

•Given the following hydrograph and the 2S/Δt + O curve, find the outflow hydrograph for the reservoir assuming it to be completely full at the beginning of the storm.
•The following hydrograph is given:

Hydrograph For Modified Puls Example
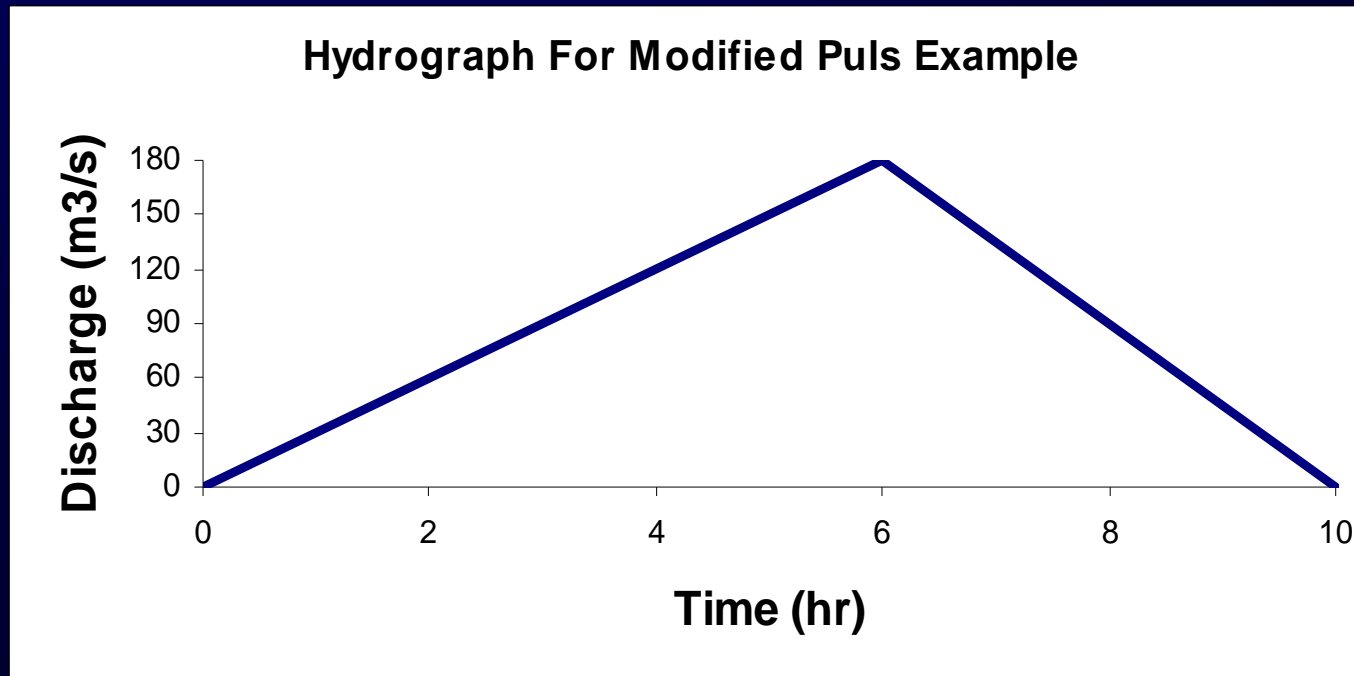
Modified Puls Example

• The following 2S/Δt + O curve is also given:

2S/Δt + O curve for Modified Puls example

Muskingum Method

$$Sp = K \, O \qquad \text{Prism Storage}$$

$$Sw = K(I - O)X \qquad \text{Wedge Storage}$$

$$S = K[XI + (1-X)O] \qquad \text{Combined}$$

Muskingum, cont...

Substitute storage equation, S into the "S" in the continuity equation yields :

$$S = K[XI + (1-X)O]$$

$$\frac{dS}{dt} = I - O$$

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

$$C_0 = -\frac{Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

Muskingum Notes :

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- The method assumes a single stage-discharge relationship.

- In other words, for any given discharge, Q, there can be only one stage height.

- This assumption may not be entirely valid for certain flow situations.

- For instance, the friction slope on the rising side of a hydrograph for a given flow, Q, may be quite different than for the recession side of the hydrograph for the same given flow, Q.

- This causes an effect known as hysteresis, which can introduce errors into the storage assumptions of this method.

Estimating K

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- K is estimated to be the travel time through the reach.

- This may pose somewhat of a difficulty, as the travel time will obviously change with flow.

- The question may arise as to whether the travel time should be estimated using the average flow, the peak flow, or some other flow.

- The travel time may be estimated using the kinematic travel time or a travel time based on Manning's equation.

Estimating X

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Wave
Muskingum
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Cunge
Dynamic
Modeling
Notes

- The value of X must be between 0.0 and 0.5.

- The parameter X may be thought of as a weighting coefficient for inflow and outflow.

- As inflow becomes less important, the value of X decreases.

- The lower limit of X is 0.0 and this would be indicative of a situation where inflow, I, has little or no effect on the storage.

- A reservoir is an example of this situation and it should be noted that attenuation would be the dominant process compared to translation.

- Values of X = 0.2 to 0.3 are the most common for natural streams; however, values of 0.4 to 0.5 may be calibrated for streams with little or no flood plains or storage effects.

- A value of X = 0.5 would represent equal weighting between inflow and outflow and would produce translation with little or no attenuation.

Did you know?

Lag and K is a special case of
Muskingum -> X=0!

More Notes - Muskingum

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- The Handbook of Hydrology (Maidment, 1992) includes additional cautions or limitations in the Muskingum method.

- The method may produce negative flows in the initial portion of the hydrograph.

- Additionally, it is recommended that the method be limited to moderate to slow rising hydrographs being routed through mild to steep sloping channels.

- The method is not applicable to steeply rising hydrographs such as dam breaks.

- Finally, this method also neglects variable backwater effects such as downstream dams, constrictions, bridges, and tidal influences.

Muskingum Example Problem

- A portion of the inflow hydrograph to a reach of channel is given below. If the travel time is K=1 unit and the weighting factor is X=0.30, then find the outflow from the reach for the period shown below:

Time	Inflow	C_0I_2	C_1I_1	C_2O_1	Outflow
0	3				3
1	5				
2	10				
3	8				
4	6				
5	5				

Muskingum Example Problem

- The first step is to determine the coefficients in this problem.
- The calculations for each of the coefficients is given below:

$$C_0 = - \frac{Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$C_0 = - ((1*0.30) - (0.5*1)) / ((1-(1*0.30) + (0.5*1)) = 0.167$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$C_1 = ((1*0.30) + (0.5*1)) / ((1-(1*0.30) + (0.5*1)) = 0.667$

Muskingum Example Problem

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

C_2= (1- (1*0.30) - (0.5*1)) / ((1-(1*0.30) + (0.5*1)) = 0.167

Therefore the coefficients in this problem are:
- C_0 = 0.167
- C_1 = 0.667
- C_2 = 0.167

Muskingum Example Problem

- The three columns now can be calculated.
- $C_0 I_2 = 0.167 * 5 = 0.835$
- $C_1 I_1 = 0.667 * 3 = 2.00$
- $C_2 O_1 = 0.167 * 3 = 0.501$

Time	Inflow	$C_0 I_2$	$C_1 I_1$	$C_2 O_1$	Outflow
0	3	0.835	2.00	0.501	3
1	5				
2	10				
3	8				
4	6				
5	5				

Muskingum Example Problem

- Next the three columns are added to determine the outflow at time equal 1 hour.

- $0.835 + 2.00 + 0.501 = 3.34$

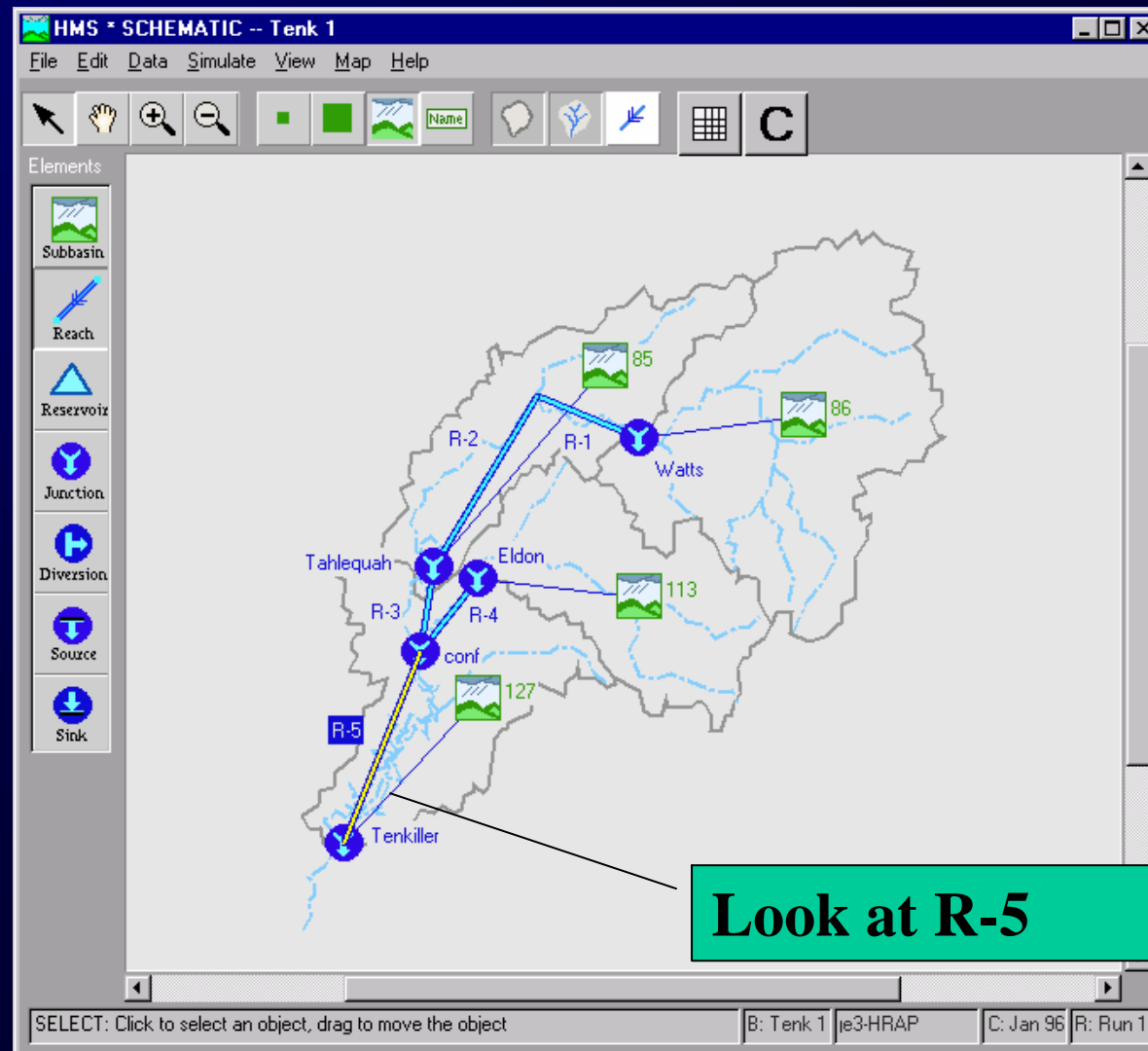| Time	Inflow	C_0I_2	C_1I_1	C_2O_1	Outflow
0	3	0.835	2.00	0.501	3
1	5				3.34
2	10				
3	8				
4	6				
5	5				

Muskingum Example Problem

- This can be repeated until the table is complete and the outflow at each time step is known.

Time	Inflow	C_0I_2	C_1I_1	C_2O_1	Outflow
0	3	0.835	2.00	0.501	3
1	5	1.67	3.34	0.557	3.34
2	10	1.34	6.67	0.93	5.57
3	8	1.00	5.34	1.49	8.94
4	6	0.835	4.00	1.31	7.83
5	5		3.34	1.03	6.14

Muskingum Example - Tenkiller

Full Dynamic Wave Equations

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- The solution of the St. Venant equations is known as dynamic routing.

- Dynamic routing is generally the standard to which other methods are measured or compared.

- The solution of the St. Venant equations is generally accomplished via one of two methods : 1) the method of characteristics and 2) direct methods (implicit and explicit).

- It may be fair to say that regardless of the method of solution, a computer is absolutely necessary as the solutions are quite time consuming.

- J. J. Stoker (1953, 1957) is generally credited for initially attempting to solve the St. Venant equations using a high speed computer.

Dynamic Wave Solutions

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- Characteristics, Explicit, & Implicit

- The most popular method of applying the implicit technique is to use a four point weighted finite difference scheme.

- Some computer programs utilize a finite element solution technique; however, these tend to be more complex in nature and thus a finite difference technique is most often employed.

- It should be noted that most of the models using the finite difference technique are one-dimensional and that two and three-dimensional solution schemes often revert to a finite element solution.

Dynamic Wave Solutions

Modified Puls
Kinematic Wave
Muskingum
Muskingum-Cunge
Dynamic
Modeling Notes

- Dynamic routing allows for a higher degree of accuracy when modeling flood situations because it includes parameters that other methods neglect.

- Dynamic routing, when compared to other modeling techniques, relies less on previous flood data and more on the physical properties of the storm. This is extremely important when record rainfalls occur or other extreme events.

- Dynamic routing also provides more hydraulic information about the event, which can be used to determine the transportation of sediment along the waterway.

Courant Condition?

Modified Puls
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Muskingum
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Dynamic
Modeling Notes

- If the wave or hydrograph can travel through the subreach (of length Δx) in a time less than the computational interval, Δt, then computational instabilities may evolve.

- The condition to satisfy here is known as the Courant condition and is expressed as :

$$dt \leq \frac{dx}{c}$$

Some DISadvantages

Modified Puls
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Muskingum
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Dynamic
Modeling Notes

- Geometric simplification - some models are designed to use very simplistic representations of the cross-sectional geometry. This may be valid for large dam breaks where very large flows are encountered and width to depth ratios are large; however, this may not be applicable to smaller dam breaks where channel geometry would be more critical.

- Model simulation input requirements - dynamic routing techniques generally require boundary conditions at one or more locations in the domain, such as the upstream and downstream sections. These boundary conditions may in the form of known or constant water surfaces, hydrographs, or assumed stage-discharge relationships.

- Stability - As previously noted, the very complex nature of these methods often leads to numeric instability. Also, convergence may be a problem in some solution schemes. For these reasons as well as others, there tends to be a stability problem in some programs. Often times it is very difficult to obtain a "clean" model run in a cost efficient manner.